A Locking-free Kriging-based Timoshenko Beam Element with the Discrete Shear Gap Technique

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The Kriging-based FEM (K-FEM) is an enhancement of the FEM through the use of Kriging interpolation in place of the conventional polynomial interpolation. It was firstly proposed a decade ago with the application to the 2D elastostatic problem. The Kriging interpolation is constructed for each element using the nodal values at the nodes within its own element as well as several layers of surrounding elements. The element and the several-layer of surrounding elements constitute the domain of influencing nodes (DOI) for the element interpolation. With the K-FEM, high degree shape functions can be easily constructed. Very accurate and smooth solutions can be achieved even using mesh of simple elements such as three-node triangles in 2D problems and four-node tetrahedrons in 3D problems. Furthermore, the formulation and computer implementation of the K-FEM are very similar to the conventional FEM so that it can be incorporated into an existing FEM code without a major change. The shortcoming of the K-FEM is that the interpolation function is discontinuous across the element boundaries. Despite the nonconformity, the results of K-FEM with appropriate Kriging parameters converge well to the corresponding exact solutions.

In the development of the K-FEM for analyses of shear deformable plates and shells, the well-known difficulty of shear locking also presents. In the previous works, the locking in the K-FEM for plates and shells were alleviated using a sufficiently high degree polynomial basis in the Kriging interpolation, i.e. the cubic or quartic basis. However, the use of a high degree polynomial interpolation can only *alleviate*, but cannot *eliminate* the locking completely. In the simpler context of the Kriging-based Timoshenko beam element (K-Beam) the shear locking can be easily eliminated using the selective reduced integration technique. This technique, however, is not applicable to the Kriging-based Reissner-Mindlin triangular plate bending element (K-Plate). Thus a technique to eliminate the shear locking in the K-Plate needs to be developed.

A unified approach for construction of locking-free finite elements for shear deformable plates and shells, called the discrete shear gap (DSG) technique, was considered as a possibility to eliminate the shear locking in the K-Plate. The key advantage of this approach is that it is based on pure displacement based formulation, needs only the usual deflection and rotational degrees of freedom at the nodes, and is directly applicable to triangular and quadrilateral elements of arbitrary polynomial degree. Prior to applying the DSG technique to the K-Plate, it is instructive to study its application in the simpler context of Timoshenko beam model. Thus our aim here is to present the development of the K-Beam with the DSG technique.

In this development, the variational form of the governing equations for static, free vibration, and buckling analyses of the beam were derived using the Hamilton's principle. The shape functions for the beam deflection and rotation were constructed using the aforementioned pure displacement Kriging-based finite element procedure. In order to eliminate the shear locking, the interpolation for the transverse shear strain was then modified using the DSG technique. The numerical tests show that the DSG technique can completely eliminate the shear locking for the Kriging-based beam element with cubic basis and three-layer DOI. Highly accurate results were obtained not only for the displacement, natural frequencies, buckling load, bending moment, but also for the shear stress along the beam. We found, however, that the DSG technique does not work well for the K-beam with linear and quadratic bases. The successful application of the DSG technique in the Timoshenko beam model motivates us to extend it to the Reissner-Mindlin plate model, which is now still ongoing research.